To show that partial molar quantities calculated using the Van Nes equation satisfy the Gibbs-Duhem equation, we'll break down the derivation step by step.

Let's start with the Van Nes equation:

Xᵢ = (dG/dnᵢ)ₚ, T

where Xᵢ represents the partial molar quantity of component I in a mixture, G is the Gibbs free energy of the mixture, nᵢ is the amount of component I, and the subscript p, T denotes the partial molar quantity at constant pressure and temperature.

The Gibbs-Duhem equation relates the partial molar quantities in a mixture and is given by:

Σ(xᵢdμᵢ) = 0

where xᵢ represents the mole fraction of component I in the mixture, μᵢ represents the chemical potential of component I, and the summation is taken over all components in the mixture.

Now, let's proceed with the derivation:

Step 1: Take the differential of the Van Nes equation.

We start by taking the differential of the Van Nes equation to express the change in partial molar quantity (dXᵢ) in terms of changes in variables:

dXᵢ = d((dG/dnᵢ)ₚ, T)

Step 2: Applying the chain rule.

Next, we use the chain rule to expand the differential on the right-hand side:

dXᵢ = (∂²G/∂nᵢ∂nⱼ)ₚ,T dnⱼ + (∂²G/∂nᵢ∂T)ₚ,dnᵢ + (∂²G/∂nᵢ²)ₚ,T(dnᵢ)²

Step 3: Expressing partial derivatives as chemical potential derivatives.

We rewrite the terms in the expanded expression using the definitions of partial derivatives:

dXᵢ = (∂²G/∂nᵢ∂nⱼ)ₚ,T dnⱼ + (∂μᵢ/∂T)ₚ,dnᵢ + (∂μᵢ/∂nᵢ)ₚ,T(dnᵢ)²

Here, (∂²G/∂nᵢ∂nⱼ)ₚ, T represents the second partial derivative of the Gibbs free energy concerning the mole numbers of components i and j, at constant pressure and temperature. Similarly, (∂μᵢ/∂T)ₚ,dnᵢ represents the partial derivative of the chemical potential μᵢ concerning temperature T, at constant pressure p and mole number nᵢ. And (∂μᵢ/∂nᵢ)ₚ, T represents the partial derivative of the chemical potential μᵢ concerning the mole number nᵢ, at constant pressure p and temperature T.

Step 4: The Gibbs-Duhem equation

Now, let's consider the Gibbs-Duhem equation:

Σ(xᵢdμᵢ) = 0

We take the differential of both sides to analyze the change in the summation:

d(Σ(xᵢdμᵢ)) = 0

Expanding the summation:

Σ(xᵢdμᵢ) + Σ(μᵢdxᵢ) = 0

Rearranging the terms:

Σ(μᵢdxᵢ) = - Σ(xᵢdμ

ᵢ)

Step 5: Comparing the expressions.

Comparing the expression for dXᵢ from Step 3 and the rearranged expression from Step 4, we observe that the terms Σ(μᵢdxᵢ) and -Σ(xᵢdμᵢ) are equivalent.

Therefore, we conclude that:

dXᵢ = Σ(μᵢdxᵢ)

which is consistent with the Gibbs-Duhem equation.

This demonstrates that the partial molar quantities calculated using the Van Nes equation satisfy the Gibbs-Duhem equation. The derivation shows that the differential of the partial molar quantity, as obtained from the Van Ness equation, is equal to the sum of the products of the chemical potentials and differentials of mole fractions, which precisely matches the Gibbs-Duhem equation.