**TOPOLOGICAL SPACES**

**NAME**

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TOPOLOGICAL SPACES

Topological spaces are mathematical structures that define abstract relations of convergence, continuity, and connectedness between objects in terms of relationships between sets.

*Definition 1:* Let X be a set and C be a collection of subsets of X. Where C is called the topology of X, elements of X are points. Then (X, C) is the topological space, and element C is an open set of X if it satisfies**:-**

* The empty set (Ø) and X are in C.
* The intersection of any finite collection of sets in C is in C.
* An arbitrary (finite or infinite) union of sets C belongs to C.

**Example:** X = {1, 2, 3, 4}, and the collection T=F(x) forms a discrete topology. (Where F(x) is a power set of X.)

Or let X be a set and C a collection of subsets of X. Then (X, C) is a topological space, and C is a closed set of X if it satisfies:-

* The intersection of an arbitrary number of sets in C is in C.
* The union of a finite collection of sets in C is in C.

Under this definition, sets in the topology C are closed sets, and their complements in X are open sets. A topological space in which points are functions is called a function space. The Kuratowski closure axioms are a set of axioms that can be used to define topological structures on a set. Lipp, Johannes; Weisstein, EricW. (2023).

*Definition 2:* In terms of neighbourhoods, some axioms relating sets, points, and their neighbourhood which must follow the following list:

* Every point belongs to every one of its neighbourhood if T is a neighbourhood of n where n belongs to T.
* Suppose T is a set of N and that T includes a neighbourhood of n in N. Then T is a neighbourhood of n.
* If ϕ is an intersection of two neighbourhoods of n, then ϕ is a neighbourhood of N.
* Every neighbourhood of T of n also includes a neighbourhood S of n, and T is a neighbourhood of each point of S.

*Definition 3:* A topological space is known as a compact space or bicompact space if every open set U of X contains a finite subset V such that V⊂U.

A topological space X becomes a paracompact space if for every open set U of X there is a finite open set V such that V<U1.

* Let ⨍ be a continuous mapping of a compact space X onto a topological space Z. Then Z is compact.

*Proof:* let V be an open set of Z, then

⨍-1(V) = {⨍-1(V) |VϵV} is an open set of X. Since X is compact, there is a finite subset {⨍-1(V1),…, ⨍-1(Vk)} of ⨍-1(V). Then (V1, V2,…, Vk) is a finite subset of V, proving the compactness of Z.

* Every closed set of a compact space is compact. i.e. T2-space is closed.

*Proof:* let A be a non-closed set of a T2space X. Then there is a point ρϵ Ᾱ-A. For each point q ϵ A, take an open neighbourhood U(q) of q and neighborhood Vq(p) of p such that U(q)⨅ Vq(p) = Ø. Then U = {A ⨅ U(q) | q ϵ A } is an open set of A.

* Let X be a compact space and Y a T2- space then every continuous one-to-one mapping of X onto Y is a topological mapping. D.Lazard (2022). Wikipedia

**Continuous functions –** A function f: X→Y between topological spaces is continuous if, for every x ϵ X and every neighborhood T of f(x), there is neighborhood S of x such that f(M)⊆N. Also, f is continuous if the inverse image of every open set is open.

Homeomorphism is a bijection that is continuous and its inverse is continuous too.

Topological spaces include:

1. Discrete topology (in which every subset is open)
2. Trivial topology (in which only empty set and whole space are open. Here every sequence and net converge to every point of the space)
3. Hausdorff spaces (where every limit points are unique)

GIS wiki from GIS Encyclopaedia. (2011)

**REFERENCE**

Lipp, Johannes, Weisstein, Eric.W. (2023). Wolfram Math World

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