**TOPOLOGICAL SPACE.**

**Introduction.**

Topological space is a set whose elements are called points, along with an additional structure called topology. It can also be defined as a set of neighborhoods for each point that satisfy some axioms formalizing the concept of closeness.

It provides a general framework for study of convergence, continuity and compactness. The fundamental structure on topological space is not a distance function, but a collection of open sets; thinking directly in terms of open sets often leads to greater clarity as well as greater generality.

**Definition 1.1**

A topology on a nonempty set X is a collection of subsets of X, called open sets, such that:

1. The empty set ø and the set X are open.
2. The union of an arbitrary collection of open sets is open;
3. The intersection of a finite number of open sets is open.

A subset A of X is closed set if and only if its compliment, Ac = X\A, is open.

More formally, a collection Tof subsets of X is a topology of X if:

1. Ø,X € T:
2. If Ga € T for a € A, then Ua€AGa€T;
3. If Gi € i = 1,2….,n, then Ưi =1Gi € T

We call the pair (X,T) a topological space; if t is clear from,then we often refer to X as a topological space.

**Definition 1.2**

Let X be a nonempty set. The collection {ø,X}, consisting of the empty set and the whole set, is a topology on X, called the trivial topology or induces topology. The power set p(X) 0f X, is a topologyon X, called the discrete topology.

**Definition 1.3**

Let (X,d) be metric space. Then the set of all open sets is a topology on X, called a metric topology.

**Definition 1.4**

Let (X,T) be a topological space and y a sunset of X. Then

 S = {H⊂Y| H = G∩ for some G € T} is a topology on Y. The open sets in Y are the intersections of open sets in X with Y. This topology is called a topological subspace of (X,T). For instance, the interval [0,1/2]is an open subset of [0,1] with respect to induced metric topology of [0,1]in R, since [0, ½] = (-1/2,1/2)∩[0,1].

**Definition 1.5**

A sequence (xn) in X conveges to a limit x € X if for every neighborhood V of x, there is a number N such that xn € for all n≥ N.

This definition says that the sequecnce eventually lies entirely in every neighborhood of x.

**Definition 1.6**

A function f:X → Y is continuous at x € X if for each neighborhood W of f(x) there exists a neighborhood V of x such that f(V) ⊂ W. we say that f is continuous on X if and only if it is continuous at every x € X.

**Definition 1.7**

Let (X,T) and (Y,S) be two topological spaces and f : X → Y. Then f is comtinous on X if and only if f-1(G) € T for every G € S.

Thus , a continuous fuction is characterized by the property that the inverse image of an open set ius open.

**Definition 1.8**

A function f : X → Y between topological spaces X and Y is a homeomorphism if it is a one- to-one, onto map and both f and f-1 are continuous . Two topological spaces X and Y are homeomorphism f : X → Y.

Homoemorphic spaces are indistinguishable as topological spaces. For example, if f : X → Y.is a homeomorphism, then G is open in X if and only if f(G) is open in Y, and a sequence(f (xn)) converges to f(x) in Y.

A one-to-one , onto map f always has an inverse f-1but f-1 need not to be continouseven if f is.

**Definition 1.9**

We define f: [0,2π) → T by f(θ) = eiθ, where [0,2π) ⊂ R with the topology induced by the usual topology on R, and T ⊂ C is a unit circle with the topology induced by the usual topology on C. Then , f is is continuous but f-1 is not.

**Definition 1.10**

A subset k of a topological space X is compact if every open cover of k contains a finite subcover.

It follows from the definition that a subset K of X is compact in the topology on X if and only if K is compact as a subset of itself with respect to the relative topology of K in X. This contrasts with the fact that a set G ⊂ Y may be relatively open in Y, yet not be open in X. For this reason, while we define the notion of relatively open, we do not define the notion of relatively compact.