**MATHEMATICS TASK**

**TITLE : THE STOCHASTIC PROPERTIES IN STASTICAL MATHEMATICS**

**INTRODUCTION**

A stochastic process is defined as a collection of random variables defined on a common [probability space](https://en.wikipedia.org/wiki/Probability_space) (Ω)

In [probability theory](https://en.wikipedia.org/wiki/Probability_theory) and related fields, a stochastic or random process is a [mathematical object](https://en.wikipedia.org/wiki/Mathematical_object) usually defined as a [sequence](https://en.wikipedia.org/wiki/Sequence) of [random variables](https://en.wikipedia.org/wiki/Random_variable), where the [index](https://en.wikipedia.org/wiki/Index_set) of the sequence has the interpretation of [time](https://en.wikipedia.org/wiki/Time).

 [Stochastic](https://en.wikipedia.org/wiki/Stochastic) processes are widely used as [mathematical models](https://en.wikipedia.org/wiki/Mathematical_model) of systems and phenomena that appear to vary in a random manner. Examples include the growth of a [bacterial](https://en.wikipedia.org/wiki/Bacteria) population, an [electrical current](https://en.wikipedia.org/wiki/Electrical_current) fluctuating due to [thermal noise](https://en.wikipedia.org/wiki/Thermal_noise), or the movement of a [gas](https://en.wikipedia.org/wiki/Gas) [molecule](https://en.wikipedia.org/wiki/Molecule).

Stochastic processes have applications in many disciplines such as [biology](https://en.wikipedia.org/wiki/Biology),  [chemistry](https://en.wikipedia.org/wiki/Chemistry) ,[neuroscience](https://en.wikipedia.org/wiki/Neuroscience), [physics](https://en.wikipedia.org/wiki/Physics), [image processing](https://en.wikipedia.org/wiki/Image_processing), [signal processing](https://en.wikipedia.org/wiki/Signal_processing), [control theory](https://en.wikipedia.org/wiki/Stochastic_control), I[nformation theory](https://en.wikipedia.org/wiki/Information_theory), [computer science](https://en.wikipedia.org/wiki/Computer_science), and [telecommunications](https://en.wikipedia.org/wiki/Telecommunications).

A stochastic process can be classified in different ways, for example, by its state space, its index set, or the dependence among the random variables. One common way of classification is by the [cardinality](https://en.wikipedia.org/wiki/Cardinality) of the index set and the state space.

When interpreted as time, if the index set of a stochastic process has a finite or countable number of elements, such as a finite set of numbers, the set of integers, or the natural numbers, then the stochastic process is said to be in [discrete time](https://en.wikipedia.org/wiki/Discrete_time). If the index set is some interval of the real line, then time is said to be [continuous](https://en.wikipedia.org/wiki/Continuous_time). The two types of stochastic processes are respectively referred to as discrete-time and [continuous-time stochastic processes](https://en.wikipedia.org/wiki/Continuous-time_stochastic_process).

 Discrete-time stochastic processes are considered easier to study because continuous-time processes require more advanced mathematical techniques and knowledge,

If the state space is the integers or natural numbers, then the stochastic process is called a discrete  stochastic process. If the state space is the real line, then the stochastic process is referred to as a real-valued stochastic process.

**THE MAIN EXAMPLES OF STOCHASTIC PROCESSES**

In stochastic process, we have six main examples namely;

1. Bernoulli process
2. Random walk
3. Wiener process
4. Poisson
5. Markov processes and chains
6. Martingales

**DISCUSSION ON THE PROCESSES**

1. **Bernoulli process**

One of the simplest stochastic processes is the [Bernoulli process](https://en.wikipedia.org/wiki/Bernoulli_process), which is a sequence of [independent and identically distributed](https://en.wikipedia.org/wiki/Independent_and_identically_distributed) (iid) random variables, where each random variable takes either the value one or zero, say one with probability and zero with probability. This process can be linked to repeatedly flipping a coin, where the probability of obtaining a head is and its value is one, while the value of a tail is zero. In other words, a Bernoulli process is a sequence of iid Bernoulli random variables, where each coin flip is an example of a [Bernoulli trial](https://en.wikipedia.org/wiki/Bernoulli_trial).

The Bernoulli trials process, named after Jacob Bernoulli, is one of the simplest yet most important random processes in probability. Essentially, the process is the mathematical abstraction of coin tossing, but because of its wide applicability, it is usually stated in terms of a sequence of generic trials.

A sequence of Bernoulli trials satisfies the following assumptions:

1. Each trial has two possible outcomes, in the language of reliability called success and failure.
2. The trials are independent. Intuitively, the outcome of one trial has no influence over the outcome of another trial.
3. On each trial, the probability of success is p and the probability of failure is 1−p − where p∈ [0,1] is the success parameter of the process.

Mathematically, we can describe the Bernoulli trials process with a sequence of indicator random variables:

X=(X1,X2,…)(11.1.1)(11.1.1)=(1,2,…)

An indicator variable is a random variable that takes only the values 1 and 0, which in this setting denote success and failure, respectively. Indicator variable Xi simply records the outcome of trial i. Thus, the indicator variables are independent and have the same probability density function:

The joint probability density function of (X1,X2,…,Xn)(1,2,…,) trials is given by

fn(x1,x2,…,xn)=px1+x2+⋯+xn(1−p)n−(x1+x2+⋯+xn),(x1,x2,…,xn)∈{0,1}n(11.1.3)(11.1.3)(1,2,…,)=1+2+⋯+(1−)−(1+2+⋯,…,)∈{0,1}

1. **Random walk**

[*Random walks*](https://en.wikipedia.org/wiki/Random_walks) are stochastic processes that are usually defined as sums of [iid](https://en.wikipedia.org/wiki/Iid%22%20%5Co%20%22Iid) random variables or random vectors in Euclidean space, so they are processes that change in discrete time. But some also use the term to refer to processes that change in continuous time, particularly the Wiener process used in finance, which has led to some confusion, resulting in its criticism. There are other various types of random walks, defined so their state spaces can be other mathematical objects, such as lattices and groups, and in general they are highly studied and have many applications in different disciplines.

A classic example of a random walk is known as the *simple random walk*, which is a stochastic process in discrete time with the integers as the state space, and is based on a Bernoulli process, where each Bernoulli variable takes either the value positive one or negative one. In other words, the simple random walk takes place on the integers, and its value increases by one with probability.

A stochastic process {X}n∈N0{ ∈0 is said to be a **simple symmetric random walk (SSRW)** if

1. X0=0 ,
2. the random variables δ1=X1−X0 , δ2=X2−X1 , …, called the **steps** of the random walk, are independent
3. each δn has a **coin-toss distribution**, i.e., its distribution is given byP[δn=1]=P[δn=−1]=12 for each n.

Some comments:

* This definition captures the main features of an idealized notion of a particle that gets shoved, randomly, in one of two possible directions, over and over. In other words, these “shoves” force the particle to take a step, and steps are modeled by the random variables variables δ1,δ2,…,…. The position of the particle after n steps is Xn; indeed,Xn=δ1+δ2+⋯+δn for n∈N.1+2+⋯+ for ∈.It is important to assume that any two steps are independent of each other - the most important properties of random walks depend on this in a critical way.
* Sometimes, we only need a finite number of steps of a random walk, so we only care about the random variables X0,X1,…,XT 0,1,…,. This stochastic process (now with a finite time horizon T) will also be called a random walk. If we want to stress that the horizon is not infinite, we sometimes call it the **finite-horizon random walk**. Whether T is finite or infinite is usually clear from the context.
* The starting point X0=0 is just a normalization. Sometimes we need more flexibility and allow our process to start at X0=0 for some x∈N To stress that fact, we talk about the random walk **starting at**x. If no starting point is mentioned, you should assume X0=0
1. **Wiener process**

The Wiener process is a stochastic process with [stationary](https://en.wikipedia.org/wiki/Stationary_increments) and [independent increments](https://en.wikipedia.org/wiki/Independent_increments) that are [normally distributed](https://en.wikipedia.org/wiki/Normally_distributed) based on the size of the increments. The Wiener process is named after [Norbert Wiener](https://en.wikipedia.org/wiki/Norbert_Wiener), who proved its mathematical existence, but the process is also called the Brownian motion process or just Brownian motion due to its historical connection as a model for [Brownian movement](https://en.wikipedia.org/wiki/Brownian_movement) in liquids.



Realizations of Wiener processes (or Brownian motion processes) with drift (blue) and without drift (red).

Playing a central role in the theory of probability, the Wiener process is often considered the most important and studied stochastic process, with connections to other stochastic processes. Its index set and state space are the non-negative numbers and real numbers, respectively, so it has both continuous index set and states space. But the process can be defined more generally so its state space can be dimensional Euclidean space. If the [mean](https://en.wikipedia.org/wiki/Mean) of any increment is zero, then the resulting Wiener or Brownian motion process is said to have zero drift. If the mean of the increment for any two points in time is equal to the time difference multiplied by some constant, which is a real number, then the resulting stochastic process is said to have drift

[Almost surely](https://en.wikipedia.org/wiki/Almost_surely), a sample path of a Wiener process is continuous everywhere but [nowhere differentiable](https://en.wikipedia.org/wiki/Nowhere_differentiable_function). It can be considered as a continuous version of the simple random walk. The process arises as the mathematical limit of other stochastic processes such as certain random walks rescaled, which is the subject of [Donsker's theorem](https://en.wikipedia.org/wiki/Donsker%27s_theorem%22%20%5Co%20%22Donsker%27s%20theorem) or invariance principle, also known as the functional central limit theorem.

The Wiener process is a member of some important families of stochastic processes, including Markov processes, Lévy processes and Gaussian processes. The process also has many applications and is the main stochastic process used in stochastic calculus. It plays a central role in quantitative finance, where it is used, for example, in the Black–Scholes–Merton model. The process is also used in different fields, including the majority of natural sciences as well as some branches of social sciences, as a mathematical model for various random phenomena.

1. **Poisson process**

The Poisson process is a stochastic process that has different forms and definitions. It can be defined as a counting process which is a stochastic process that represents the random number of points or events up to some time. The number of points of the process that are located in the interval from zero to some given time is a Poisson random variable that depends on that time and some parameter. This process has the natural numbers as its state space and the non-negative numbers as its index set. This process is also called the Poisson counting process, since it can be interpreted as an example of a counting process.

If a Poisson process is defined with a single positive constant, then the process is called a homogeneous Poisson process. The homogeneous Poisson process is a member of important classes of stochastic processes such as Markov processes and Lévy processes.

The homogeneous Poisson process can be defined and generalized in different ways. It can be defined such that its index set is the real line, and this stochastic process is also called the stationary Poisson process. If the parameter constant of the Poisson process is replaced with some non-negative integrable function of �, the resulting process is called an inhomogeneous or nonhomogeneous Poisson process, where the average density of points of the process is no longer constant. Serving as a fundamental process in queueing theory, the Poisson process is an important process for mathematical models, where it finds applications for models of events randomly occurring in certain time windows.

Defined on the real line, the Poisson process can be interpreted as a stochastic process, among other random objects. But then it can be defined on the dimensional Euclidean space or other mathematical spaces, where it is often interpreted as a random set or a random counting measure, instead of a stochastic process. In this setting, the Poisson process, also called the Poisson point process, is one of the most important objects in probability theory, both for applications and theoretical reasons. But it has been remarked that the Poisson process does not receive as much attention as it should, partly due to it often being considered just on the real line, and not on other mathematical spaces.

1. **Markov processes and chains**

Markov processes are stochastic processes, traditionally in [discrete or continuous time](https://en.wikipedia.org/wiki/Discrete_time_and_continuous_time), that have the Markov property, which means the next value of the Markov process depends on the current value, but it is conditionally independent of the previous values of the stochastic process. In other words, the behavior of the process in the future is stochastically independent of its behavior in the past, given the current state of the process.

The Brownian motion process and the Poisson process (in one dimension) are both examples of Markov processes in continuous time, while [random walks](https://en.wikipedia.org/wiki/Random_walk) on the integers and the [gambler's ruin](https://en.wikipedia.org/wiki/Gambler%27s_ruin) problem are examples of Markov processes in discrete time.

A Markov chain is a type of Markov process that has either discrete [state space](https://en.wikipedia.org/wiki/State_space) or discrete index set (often representing time), but the precise definition of a Markov chain varies. For example, it is common to define a Markov chain as a Markov process in either [discrete or continuous time](https://en.wikipedia.org/wiki/Continuous_and_discrete_variables) with a countable state space (thus regardless of the nature of time), but it has been also common to define a Markov chain as having discrete time in either countable or continuous state space (thus regardless of the state space).[[196]](https://en.wikipedia.org/wiki/Stochastic_process#cite_note-Asmussen2003page7-199) It has been argued that the first definition of a Markov chain, where it has discrete time, now tends to be used, despite the second definition having been used by researchers like [Joseph Doob](https://en.wikipedia.org/wiki/Joseph_Doob) and [Kai Lai Chung](https://en.wikipedia.org/wiki/Kai_Lai_Chung).

Markov processes form an important class of stochastic processes and have applications in many areas. For example, they are the basis for a general stochastic simulation method known as [Markov chain Monte Carlo](https://en.wikipedia.org/wiki/Markov_chain_Monte_Carlo), which is used for simulating random objects with specific probability distributions, and has found application in [Bayesian statistics](https://en.wikipedia.org/wiki/Bayesian_statistics).

1. **Martingales**

A martingale is a discrete-time or continuous-time stochastic process with the property that, at every instant, given the current value and all the past values of the process, the conditional expectation of every future value is equal to the current value. In discrete time, if this property holds for the next value, then it holds for all future values. The exact mathematical definition of a martingale requires two other conditions coupled with the mathematical concept of a filtration, which is related to the intuition of increasing available information as time passes. Martingales are usually defined to be real-valued, but they can also be complex-valuedor even more general.

A symmetric random walk and a Wiener process (with zero drift) are both examples of martingales, respectively, in discrete and continuous time. For a [sequence](https://en.wikipedia.org/wiki/Sequence) of [independent and identically distributed](https://en.wikipedia.org/wiki/Independent_and_identically_distributed) random variables 1,2,3,…with zero mean, the stochastic process formed from the successive partial sums 1,1+2,1+2+3,… is a discrete-time martingale. In this aspect, discrete-time martingales generalize the idea of partial sums of independent random variables.

Martingales can also be created from stochastic processes by applying some suitable transformations, which is the case for the homogeneous Poisson process (on the real line) resulting in a martingale called the *compensated Poisson process*. Martingales can also be built from other martingales. For example, there are martingales based on the martingale the Wiener process, forming continuous-time martingales.

Martingales mathematically formalize the idea of a fair game, and they were originally developed to show that it is not possible to win a fair game. But now they are used in many areas of probability, which is one of the main reasons for studying them. Many problems in probability have been solved by finding a martingale in the problem and studying it. Martingales will converge, given some conditions on their moments, so they are often used to derive convergence results, due largely to [martingale convergence theorems](https://en.wikipedia.org/wiki/Martingale_convergence_theorem).

Martingales have many applications in statistics, but it has been remarked that its use and application are not as widespread as it could be in the field of statistics, particularly statistical inference. They have found applications in areas in probability theory such as queueing theory and Palm calculusand other fields such as economics and finance.

**CONCLUSION**

Other fields of probability were developed and used to study stochastic processes, with one main approach being the theory of large deviations. The theory has many applications in statistical physics, among other fields, and has core ideas going back to at least the 1930s. Later in the 1960s and 1970s fundamental work was done by Alexander Wentzell in the Soviet Union and [Monroe D. Donsker](https://en.wikipedia.org/wiki/Monroe_D._Donsker) and [Srinivasa Varadhan](https://en.wikipedia.org/wiki/Srinivasa_Varadhan) in the United States of America, which would later result in Varadhan winning the 2007 Abel Recurrently other researches have been made on the same,2017

The theory of stochastic processes still continues to be a focus of research, with yearly international conferences on the topic of stochastic processes.

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