**Complex Fractions with Mixed Numbers and Exponents Expressed as Fractions**

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Assignment Due Date

**Complex Fractions with Mixed Numbers and Exponents Expressed as Fractions**

In the field of algebra, complex fractions, and fractional exponents are fundamental concepts that play a crucial role in problem-solving, equation manipulation, and understanding advanced mathematical concepts. Complex fractions are expressions that contain fractions within fractions or have mixed numbers in the denominator. They often arise when dealing with ratios, proportions, and rational functions. On the other hand, fractional exponents include expressing a number's root or reciprocal as an exponent. Understanding the techniques of simplifying complex fractions with mixed numbers, examples, and step-by-step solutions and utilizing exponents expressed as fractions with the different rules in fractional exponents allows for more precise mathematical calculations and facilitates deeper comprehension of the relationships between fractions, exponents, and numerical operations.

**Complex Fractions with Mixed Numbers in the Denominator**

Complex fractions with mixed numbers in the denominator involve fractions where the numerator, denominator, or both contain mixed numbers (The Organic Chemistry Tutor, 2016). For example, consider the fraction (3 ½) /( 2 ¾). They are also called compound fractions.

## **Techniques for Simplifying Complex Fractions**

### ***Clearing the Fraction in the Denominator***

One technique for simplifying complex fractions is to eliminate the mixed number in the denominator by multiplying both the numerator and denominator by the denominator's whole number part i.e. the least common divisor (LCD) (BCcampus, 2020). For example, to simplify the complex fraction (3 ½) / (2 ¾), we multiply the numerator and denominator by the LCD of the denominators. In this case, we multiply by 4.

### ***Conversion to Improper Fractions***

Another approach is to convert the mixed numbers in the numerator and denominator to improper fractions, simplifying the expression and allowing for easier calculations. For example, (3 ½) becomes (3\*2 + 1)/2 = 7/2.

### ***Cross-Multiplication Method***

The cross-multiplication method can also be used to simplify complex fractions. This involves multiplying the numerator of the first fraction by the denominator of the second fraction and vice versa, then simplifying the resulting expression.

### ***Examples and Step-by-Step Solutions***

#### **Example 1:**

Simplify the complex fraction

3/ (2 ¼)

Step 1: Convert the mixed number into an improper fraction.

The mixed number (2 ¼) can be written as an improper fraction as follows:

(2 ¼)= (2\*4+1)/4 = (9/4)

Step 2: Invert the improper fraction.

The reciprocal (or inverse) of (9/4) is (4/9)

Step 3: Multiply the numerator by the inverted fraction.

3\* (4/9) = (3/1)\*(4\*9) = (3\*4) / (1\*9)

= (12/9)

Step 4: Simplify the fraction.

The fraction (12/9) can be simplified by dividing the numerator and denominator by their greatest common divisor (GCD), which is 3:

Therefore, the simplified form of the complex fraction 3/ (2 ¼) is (4/3)

#### **Example 2**

((2/x) + (1/y))/ ((3/x) - (4/y))

To simplify this complex fraction, we can follow these steps:

Step 1: Find the least common denominator (LCD) of the fractions in the numerator and denominator. In this case, the LCD is xy.

Step 2: Multiply each term in the numerator and denominator by the LCD.

((2/x) + (1/y))/ ((3/x) - (4/y)) = ((2/x) + (1/y)) \* (xy) / ((3/x) - (4/y))\* (xy)

Step 3: Simplify each term.

((2/x)\* (xy) + (1/y) \* (xy)) / ((3/x) \* (xy) – (4/y) \* (xy))

= ((2y) + (x)) / ((3y) – (4x))

Therefore, the simplified form of the complex fraction ((2/x) + (1/y))/ ((3/x) - (4/y)) is

((2y) + (x)) / ((3y) – (4x))

## **Exponents as Fractions**

Exponentiation is the process of raising a base number to a power. For example, in the expression (2^3), 2 is the base and 3 is the exponent, resulting in 2 being raised to the power of 3, which equals 8. Fractional exponents extend the concept of exponentiation to include non-integer powers. For instance, 2^ (1/2) represents the square root of 2, and 2^(3/4) denotes the fourth root of 2 cubed(Cuemath).

## **Rules for Fractional Exponents**

We can easily multiply or divide numbers with fractional exponents by adhering to a few guidelines. Although many individuals are familiar with whole-number exponents when it comes to fractional exponents, they frequently make mistakes that can be avoided by adhering to these fractional exponent guidelines(Cuemath).

Rule 1: a1/m × a1/n = a(1/m + 1/n)

Rule 2: a1/m ÷ a1/n = a(1/m - 1/n)

Rule 3: a1/m × b1/m = (ab)1/m

Rule 4: a1/m ÷ b1/m = (a÷b)1/m

Rule 5: a-m/n = (1/a)m/n

## **Simplifying Expressions with Fractional Exponents**

The concepts of multiplication and division can be used to understand how to reduce fractional exponents. It entails simplifying the exponent into a more understandable form. 9^1/2, for instance, can be condensed to 3. Let's use a few examples to better understand how fractional exponents might be simplified.

1) Solve 3√8 = 8^(1/3)

We are aware that the number 8 can be written as the cube of two, which is provided as 8 = 23. Since the sum of the exponents in the previous example results in 3^(1/3)=1 and 3√8=8^(1/3)=2, substituting the value of 8 results in (2/3) ^(1/3) = 2.

2) Simplify (64/125) ^2/3

Both the base and the exponent in this example are expressed as fractions. 64 can be written as a cube of four, and 125 as a cube of five. The values are 64 = 43 and 125 = 53. In the example presented, substituting their values yields (43/53)2/3. The fact that both integers share the power of 3 allows us to write (43/53) ^2/3 as ((4/5)3) ^2/3, which is equivalent to (4/5)2 because 32/3=2. We currently have (4/5)2, which is 16/25. (64/125) ^2/3 = 16/25 as a result.

### ***Fractional Exponent Multiplication Using the Same Base.***

a^(1/m) a^ (1/n) = a^ (1/m + 1/n) is the generic formula for multiplying exponents with the same base. For instance, we must add the exponents before multiplying 2^2/3 and 2^3/4. So, 2/3 + 3/4 = (17/12). As a result, 2^ (2/3) divided by 2^(3/4) equals 2^(17/12).

### ***Dividing Fractional Exponents***

We write (a^(1/m)/ a^(1/n)) = a^ (1/m - 1/n) when we divide fractional exponents with different powers but the same bases. In this case, we must take the powers away and then write the difference using a common base. As an illustration,( 5^(3/4))/ (5^(1/2)) = 5^ (3/4-1/2), which equals 5^(1/4).

We write (a^(1/m)/ b^(1/m)) = (a/b) ^(1/m) to represent the division of fractional exponents with the same powers but different bases. Here, the bases are divided according to the prescribed order, and the common power is written on it. For instance, 9^(5/6) multiplied by 3^(5/6 )results in (9/3) ^(5/6), which is 3^(5/6).

### ***Negative Fractional Exponents***

Negative fractional exponents can be interpreted as taking the reciprocal and then applying the positive exponent. Here's the general rule:

a^(-m/n) = (1/a) ^(m/n)

For illustration, let's simplify 343^(-1/3). In this case, the power is -1/3 and the base is 343. The power is first given the negative sign by taking the reciprocal of the base, which is 1/343. We currently have (1/343) ^(1/3). Since 73 = 343 and 343 is the third power of 7, the phrase can be rewritten as 1/(7^3) ^(1/3). The answer is 1/7 since 3 and 1/3 cancel one another out.

## **Applications and Extensions**

### ***Real-World Applications of Complex Fractions and Fractional Exponents***

Complex fractions and fractional exponents find applications in various real-world scenarios. For instance, in finance, complex fractions can be used to calculate interest rates or determine investment returns over time. Fractional exponents are employed in scientific calculations, such as modeling exponential decay or growth in biology or physics. ("Exponential equations in science I | Math in science | Visionlearning," 2014) .Additionally, in engineering, complex fractions and fractional exponents play a role in analyzing electrical circuits and solving differential equations(Gill et al., 2018).

### ***Advanced Topics and Further Exploration***

Further exploration of complex fractions and fractional exponents can lead to advanced topics. For example, exploring complex fractions with variables in the denominator can introduce concepts like partial fraction decomposition in calculus(Lewis, 2022). Fractional exponents can also be extended to include complex numbers, leading to complex analysis and its applications in physics and engineering

### ***Related Concepts and Connections to Other Branches of Mathematics***

Complex fractions and fractional exponents are interconnected with other branches of mathematics. Complex fractions relate to concepts in algebra, such as polynomial division and rational expressions. Fractional exponents have connections to logarithms, exponential functions, and the rules of exponents ("undefined," 2020). Understanding these connections helps establish a solid foundation for further mathematical exploration.

## **Conclusion**

In conclusion, this paper has provided a comprehensive exploration of complex fractions with mixed numbers in the denominator and fractional exponents. The key concepts and techniques, such as simplifying complex fractions through various methods and understanding the properties of fractional exponents, have been thoroughly discussed. The importance of understanding these concepts has been highlighted, emphasizing their practical applications in fields like finance, science, and engineering. To further enhance understanding and go deeper into the subject, further study and exploration are encouraged, including advanced techniques for simplification, applications in complex analysis, and exploring connections to other branches of mathematics. By expanding knowledge in this area, individuals can strengthen their problem-solving skills and gain valuable insights into the broader applications of these mathematical concepts.

# References

BCcampus. (2020, January 31). 8.5 reducing complex fractions – Intermediate algebra. BCcampus Open Publishing – Open Textbooks Adapted and Created by BC Faculty. <https://opentextbc.ca/intermediatealgebraberg/chapter/8-5-reducing-complex-fractions>

Cuemath. (n.d.). Fractional exponents. <https://www.cuemath.com/algebra/fractional-exponents/>

Exponential equations in science I | Math in science | Visionlearning. (2014, December 17). Visionlearning. <https://www.visionlearning.com/en/library/Math-in-Science/62/Exponential-Equations-in-Science-I/206>

Gill, V., Modi, K., & Singh, Y. (2018). Analytic solutions of fractional differential equation associated with RLC electrical circuit. Journal of Statistics and Management Systems, 21(4), 575-582. <https://doi.org/10.1080/09720510.2018.1466966>

Lewis, M. (2022, July 2). Partial Fraction Decomposition Calculation & Rules | Expanding Partial Fractions. <https://study.com/learn/lesson/partial-fraction-decomposition-calculation-rules-expanding.html>

The Organic Chemistry Tutor. (2016, November 4). Algebra - Simplifying complex fractions [Video]. YouTube. <https://youtu.be/PpSyx-brMyg>

undefined. (2020). Pre-Calculus, Calculus, and Beyond, 363-381. <https://doi.org/10.1090/mbk/133/07>